



**NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY**

FACULTY OF HEALTH AND APPLIED SCIENCES AND NATURAL RESOURCES
DEPARTMENT OF AGRICULTURE & NATURAL RESOURCES SCIENCES

QUALIFICATION : BACHELOR OF SCIENCE IN AGRICULTURE	
QUALIFICATION CODE: 07BASA	LEVEL: 6
COURSE CODE: MTA611S	COURSE NAME: Mathematics for Agribusiness
DATE: July 2022	PAPER: Theory
DURATION: 3 Hours	MARKS: 100

SECOND OPPORTUNITY EXAMINATION QUESTION PAPER

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INSTRUCTIONS

1. Attempt ALL the questions.
2. Write clearly and neatly.
3. Number the answers clearly and correctly.

PERMISSIBLE MATERIALS

1. All written work MUST be done in blue or black ink
2. Calculators allowed
3. No books, notes and other additional aids are allowed

THIS QUESTION PAPER CONSISTS OF 6 PAGES (including this front page).

QUESTION ONE**[MARKS]**

a. Define the limit of a function at a point a . Explain how you would use the limit concept to determine whether a function is continuous at a point a . (4)

b. Suppose the profit from selling n items is known to be $P(n) = \frac{n}{2} + \sqrt{n-3}$. Find $P(7)$ and $P(28)$. (4)

c. Use interval notation to express the domain and range of the following function:

$$P(n) = \frac{n}{2} + \sqrt{n-3} \quad (4)$$

d. Suppose the production function for a food processor is represented by a quadratic function with maxima and roots at -10 and 5 . Based on this information, answer the questions below

i. Derive the mathematical equation of the production function. (3)

ii. Find the critical point of the production function you have derived in d(i). (5)

iii. Draw and label a graph that illustrates the production function. The graph must clearly show the roots, maxima, and y -intercept points of the production function. (5)

TOTAL MARKS**[25]**

QUESTION TWO**[MARKS]**

a. Given a function:

$$g(x, y) = x^2y^{-1} \quad (6)$$

Find z_{xx} , z_{yy} and z_{xy} .

b. Let $p = 100 - q^2$ be the demand function for an Agribusiness's product. Find the rate of change of price, p , per unit with respect to quantity, q . How fast is the price changing with respect to q when $q = 5$? Assume that p is in dollars. (*Hint: the rate of change implies the derivative*). (4)

c. Find:

i. $\lim_{h \rightarrow 0} \frac{(x-h)^2 - x^2}{h}$ (3)

ii. $\lim_{t \rightarrow 4} \frac{3t-12}{t-4}$ (3)

Find an equation of the tangent line to the curve (9)

$$y = \frac{4x^2 + 3}{2x - 1}$$

at $x = 1$.

TOTAL MARKS

[25]

QUESTION THREE

[MARKS]

a. Find the first derivative of the following function:

i. $f(x) = \ln(3x^4 - 5)$ (3)

ii. $g(x) = \frac{\ln x}{x^2}$ (3)

iii. $h(x) = x^2 + \log_8(x^2 + 4)$ (4)

b. Suppose a firm's production process is represented by the following function:

$$Q = 10k + 20l - 3k^2 - 4l^2 - kl$$
 (10)

Find the quantities of inputs l and k that maximize output Q .

TOTAL MARKS

[25]

QUESTION FOUR

[MARKS]

a. Find:

i. $\int \frac{7}{x} dx$ (2)

ii. $\int 3x^2(x^3 - 7)^3 dx$ (4)

iii. $\int_1^3 \frac{2x}{x^2 + 5} dx$ (4)

b. Researchers studied the average yearly income, y (in dollars), that a farmer can expect to receive with x years of education. They estimated that the rate at which income changes with respect to education is given by:

$$\frac{dy}{dx} = 100x^{\frac{3}{2}}$$
 (5)

Derive the equation that represents the average yearly income, y , as a function of education, given that $y = N\$ 28,720$ and $x = 9$. (Hint: find the antiderivative function of the differential equation. To find the constant, use the initial conditions – i.e., $y = N\$ 28,720$ and $x = 9$).

c. Suppose a firm has an order for 200 units of its product and wishes to distribute its manufacture between two of its plants, plant 1 and plant 2. Let q_1 and q_2 (10)

denote the outputs of plants 1 and 2, respectively, and suppose the total-cost function:

$$c = f(q_1, q_2) = 2q_1^2 + q_1q_2 + q_2^2 + 200$$

How should the output be distributed to minimize costs? (*Hint: the constraint faced by the firm is: $q_1 + q_2 = 200$*).

TOTAL MARKS

[25]

THE END

FORMULA

Basic Derivative Rules

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = c f'(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

Difference Rule: $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Basic Integration Rules

1. $\int a \, dx = ax + C$

2. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

3. $\int \frac{1}{x} \, dx = \ln|x| + C$

4. $\int e^x \, dx = e^x + C$

5. $\int a^x \, dx = \frac{a^x}{\ln a} + C$

6. $\int \ln x \, dx = x \ln x - x + C$

Integration by Substitution

The following are the 5 steps for using the integration by substitution method:

- Step 1: Choose a new variable u
- Step 2: Determine the value dx
- Step 3: Make the substitution
- Step 4: Integrate resulting integral
- Step 5: Return to the initial variable x

Unconstrained optimization: Multivariate functions

The following are the steps for finding a solution to an unconstrained optimization problem:

Relative maximum	Relative minimum
1. $f_x, f_y = 0$	1. $f_x, f_y = 0$
2. $f_{xx}, f_{yy} < 0$	2. $f_{xx}, f_{yy} > 0$
3. $f_{xx} \cdot f_{yy} > (f_{xy})^2$	3. $f_{xx} \cdot f_{yy} > (f_{xy})^2$

Derivative Rules for Exponential Functions

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(e^{f(x)}) = e^{f(x)} f'(x)$$

$$\frac{d}{dx}(a^{f(x)}) = \ln(a) a^{f(x)} f'(x)$$

Derivative Rules for Logarithmic Functions

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, x > 0$$

$$\frac{d}{dx} \ln(g(x)) = \frac{g'(x)}{g(x)}$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}, x > 0$$

$$\frac{d}{dx}(\log_a g(x)) = \frac{g'(x)}{g(x) \ln a}$$

Integration by Parts

The formula for the method of integration by parts is:

$$\int u \, dv = u \cdot v - \int v \, du$$

There are three steps how to use this formula:

- Step 1: identify u and dv
- Step 2: compute u and du
- Step 3: Use the integration by parts formula

Unconstrained optimization: Univariate functions

The following are the steps for finding a solution to an unconstrained optimization problem:

- Step 1: Find the critical value(s), such that:

$$f'(a) = 0$$

- Step 2: Evaluate for relative maxima or minima
 - If $f''(a) > 0 \rightarrow$ minima
 - If $f''(a) < 0 \rightarrow$ maxima

Constrained Optimization

The following are the steps for finding a solution to a constrained optimization problem using the Lagrange technique:

- Step 1: Set up the Lagrange equation
- Step 2: Derive the First Order Equations
- Step 3: Solve the First Order Equations
- Step 4: Estimate the Lagrange Multiplier

Additionally:

- If $f_{xx} \cdot f_{yy} < (f_{xy})^2$, when f_{xx} and f_{yy} have the same signs, the function is at an **inflection point**; when f_{xx} and f_{yy} have different signs, the function is at a **saddle point**.
- If $f_{xx} \cdot f_{yy} = (f_{xy})^2$, the test is **inconclusive**.